MADE EASY PUBLICATIONS

POSTAL BOOK PACKAGE 2027

ELECTRICAL ENGINEERING

CONVENTIONAL PRACTICE SETS VOLUME - I

CONTENTS

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ELECTRIC CIRCUITS

CONVENTIONAL PRACTICE SETS

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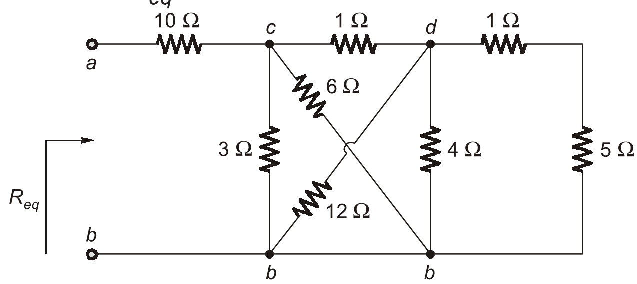
1 CHAPTER

Basics, Circuit Elements, Nodal & Mesh Analysis

Q.1 Calculate equivalent resistance R_{eq} in the circuit shown.



Solution:

3 Ω and 6 Ω resistors are in parallel because they are connected to same two nodes c and b. Their combined resistance is

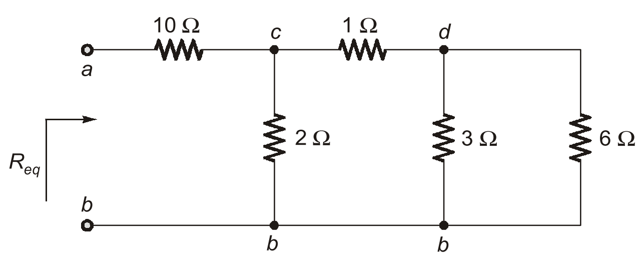
$$3\,\Omega || 4\,\Omega = \frac{3\times 6}{3+6} = 2\,\Omega$$

Similarly, 12 Ω and 4 Ω resistors are in parallel since they are connected to same two nodes *d* and *b*.

Hence,
$$12\,\Omega || 4\,\Omega = \frac{12\times 4}{12+4} = 3\,\Omega$$

Also, 1 Ω and 5 Ω resistors are in series, hence combined resistance,

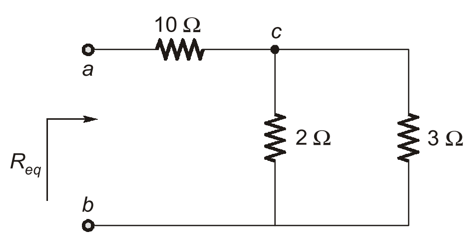
$$1\,\Omega + 5\,\Omega = 6\,\Omega$$



Further 3 Ω and 6 Ω in parallel gives equivalent resistance = $\dfrac{3\,\Omega\times 6\,\Omega}{(3+6)\,\Omega} = 2\,\Omega$

This 2 Ω is in series with 1 Ω.

Given equivalent as (2 + 1) Ω = 3 Ω as shown below.

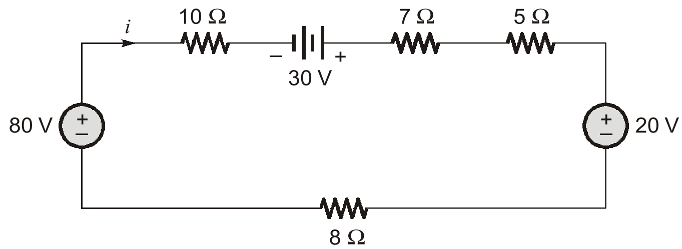


Now 2 Ω and 3 Ω parallel's combination in series with 10 Ω resistance.

Hence,
$$R_{ab} = R_{eq} = 10\,\Omega + (2\,\Omega || 3\,\Omega)$$

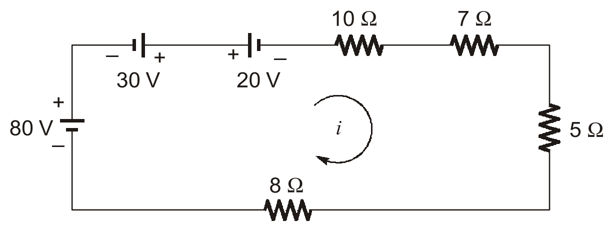
$$= 10 + \frac{2\times 3}{2+3} = 11.2\,\Omega$$

Q.2 Use resistance and source combinations to determine the current i in figure shown and power delivered by 80 V source.



Solution:

The circuit can be redrawn as,



Further combining the three voltage sources into an equivalent source of 90 V as shown below.

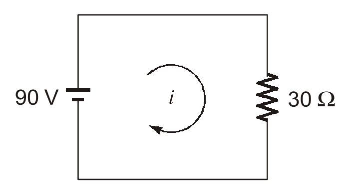
All the resistance, combined in series as,

$$R_{eq} = (10 + 7 + 5 + 8)\ \Omega = 30\ \Omega$$

Simply applying kVL, $-90 + 30i = 0$

Hence, $i = 3$ A

Power delivered by 80 V source $= 80\text{ V} \times 3\text{ A} = 240$ W



Q.3 The following mesh equations pertain to a network:

$$8I_1 - 5I_2 - I_3 = 110$$
$$-5I_1 + 10I_2 + 0 = 0$$
$$-I_1 + 0 + 7I_3 = 115$$

Draw network showing each element.

Solution:

All the mesh equations can be rearrangement as,

$$8I_1 - 5I_2 - I_3 = 110$$

$\Rightarrow$ $$5(I_1 - I_2) + (I_1 - I_3) + 2I_1 = 110 \quad ...(1)$$

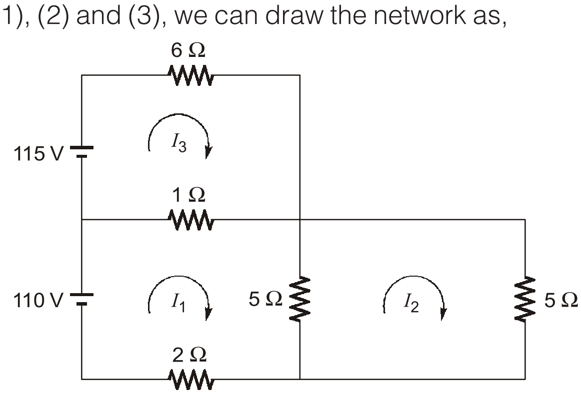
$$-5I_1 + 10I_2 + 0 = 0$$

$\Rightarrow$ $$5(I_2 - I_1) + 5I_2 = 0 \quad ...(2)$$

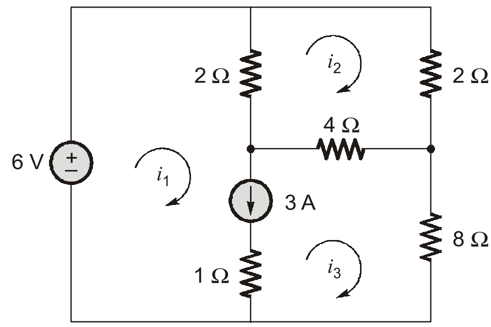
$$-I_1 + 0 + 7I_3 = 115$$

$\Rightarrow$ $$(I_3 - I_1) + 6I_3 = 115 \quad ...(3)$$

On the basis of equation (1), (2) and (3), we can draw the network as,



Q.4 Find mesh currents in the circuit,



Solution:

$$i_1 - i_3 = 3\text{ A} \quad ...(1)$$

BY KVL for super mesh,

$$2(i_1 - i_2) + 4(i_3 - i_2) + 8i_3 = 6$$
$$2i_1 - 6i_2 + 12i_3 = 6 \quad ...(2)$$

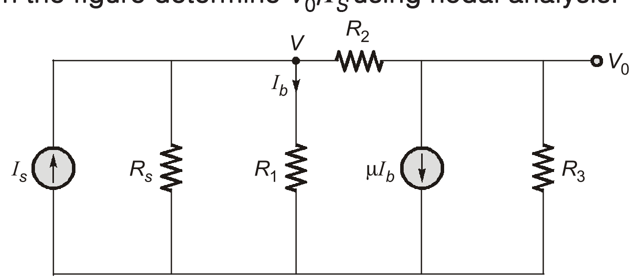
By KVL for second mesh,

$$2i_2 + 4(i_2 - i_3) + 2(i_2 - i_1) = 0$$
$$8i_2 - 4i_3 - 2i_1 = 0 \quad ...(3)$$

Solving equations (1), (2) and (3), we get

$$i_1 = 3.473\text{ A}$$
$$i_2 = 1.105\text{ A}$$
$$i_3 = 0.473\text{ A}$$

Q.5 For the circuit shown in the figure determine V_0/I_S using nodal analysis.

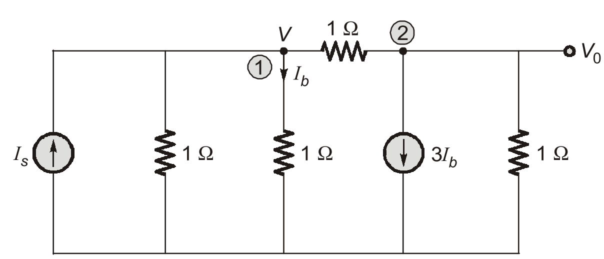


$R_1 = R_2 = R_3 = R_s = 1\ \Omega,\ \mu = 3$

Solution:

$$V = I_b \quad ...(1)$$

Node (1),



$R_1 = R_2 = R_3 = R_s = 1\ \Omega,\ \mu = 3$

$$\frac{V}{1} + \frac{V}{1} + \frac{V - V_0}{1} - I_s = 0$$
$$3V - V_0 = I_s \quad ...(2)$$

Node (2),

$$\frac{V_0}{1} + \frac{V_0 - V}{1} + 3I_b = 0$$
$$2V_0 - V = -3I_b \quad ...(3)$$

From equation (1), $I_b = V$ put in equation (3)

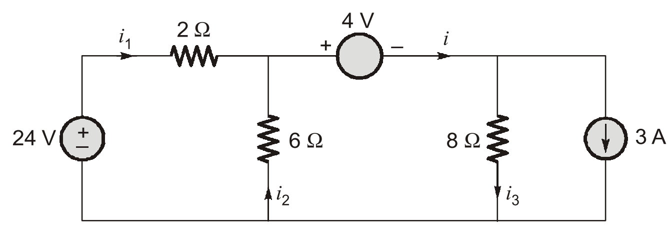
$$2V_0 - V = -3\ V$$
$$2V_0 = -2\ V$$

$\Rightarrow$ $V = -V_0$

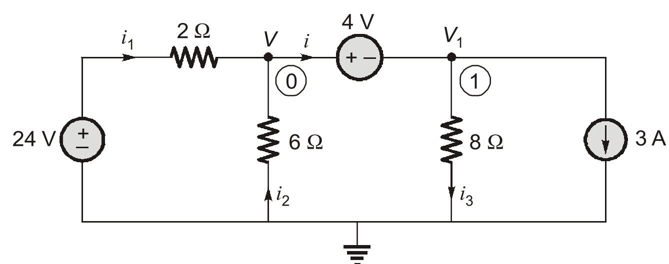
Putting, $V = -V_0$ in equation (2)

$$3(-V_0) - V_0 = I_s$$
$$-4V_0 = I_s$$
$$\frac{V_0}{I_s} = -\frac{1}{4} = -0.25$$

Q.6 For the circuit shown in figure, determine the currents i_1, i_2 and i_3 using nodal analysis.



Solution:



By nodal analysis, $-i_1 - i_2 + i = 0$

$$-\left(\frac{24-V}{2}\right) + \left[-\frac{0-V}{6}\right] + i = 0$$

$$\frac{V-24}{2} + \frac{V}{6} + i = 0 \quad ...(1)$$

$$V_1 = V - 4$$

KCL at node 1,

$$-i + \frac{V_1}{8} + 3 = 0$$

$$i = \left(\frac{V-4}{8} + 3\right) \quad ...(2)$$

Combining (1) and (2),

$$\frac{V-24}{2} + \frac{V}{6} + \frac{V-4}{8} + 3 = 0$$

Solving, $V = 12$ V

$$V_1 = 8 \text{ V}$$

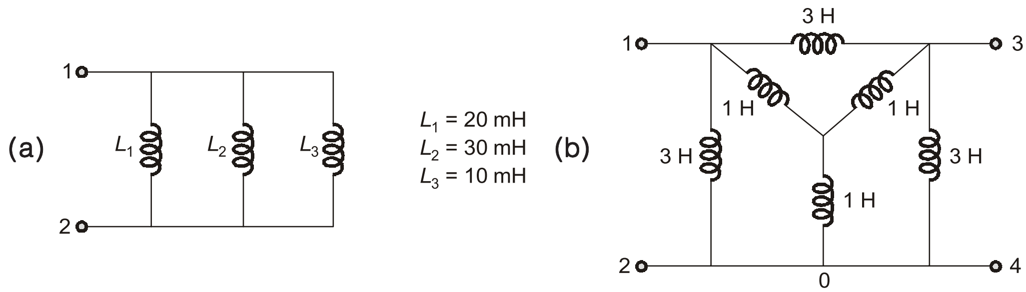
$$i_1 = \frac{24-12}{2} = 6 \text{ A}$$

$$i_2 = -\frac{12}{6} = -2 \text{ A}$$

$$i = i_3 + 3 \qquad \because i = i_1 + i_2$$

$$i_3 = i_1 + i_2 - 3$$
$$i_3 = 6 - 2 - 3 = 1 \text{ A}$$
$$i_3 = 1 \text{ A}$$

Q.7 Determine equivalent inductance at terminal '1-2' for circuits.



Solution:

(a) All the inductances are in parallel thus overall equivalent inductance is

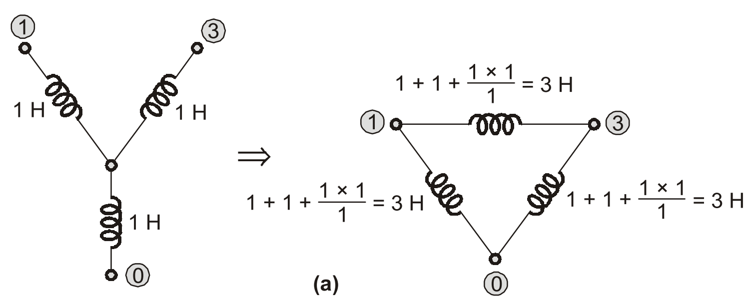
$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

$$\frac{1}{L_{eq}} = \frac{1}{20\text{ mH}} + \frac{1}{30\text{ mH}} + \frac{1}{10\text{ mH}}$$

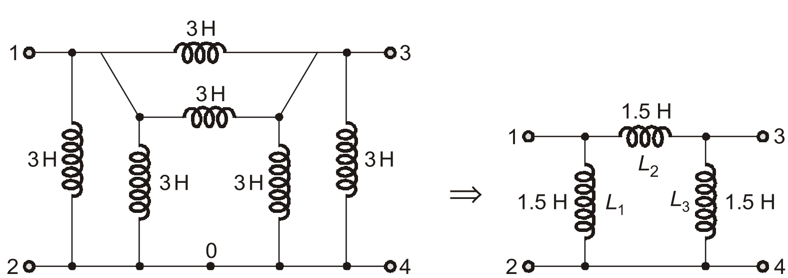
On solving, $\quad L_{eq} = \frac{60}{11}\text{ mH} = 5.45\text{ mH}$

(b) This problem can be best solved utilising star to delta transformation.

Let us first convert the interconnected inductances to an equivalent delta. This is shown in figure (a) Hence the equivalent circuit configuration of figure given becomes as shown in figure (b).



(a)



(b)

Redrawing circuits,

Thus the equivalent inductance across 1-2 is given by

$$L_{1-2} = L_1 \parallel (L_2 + L_3) = 1.5 \parallel 3 = \frac{1.5 \times 3}{1.5 + 3}.$$

Hence, $\quad L_{12} = L_{eq} = 1\text{ H}$

ELECTRICAL AND ELECTRONIC MEASUREMENTS

CONVENTIONAL PRACTICE SETS

1
CHAPTER

Errors in Measurements

Q.1 Four ammeters M1, M2, M3 and M4 with the following specifications are available:

Instrument	Type	Full scale value (A)	Accuracy % of FS
M1	$3\frac{1}{2}$ digit dual slope	20	±0.10
M2	PMMC	10	±0.20
M3	Electro-dynamic	5	±0.50
M4	Moving-iron	1	±1.00

A current of 1 A is to be measured. Calculate the error in the reading of each instruments and which meter has least error?

Solution:

Error in reading of first meter = FSD × accuracy = $20 \times \frac{\pm 0.1}{100} = \pm 0.02$

Error in reading of second meter = $10 \times \frac{\pm 0.2}{100} = \pm 0.02$

Error in reading of third meter = $5 \times \frac{\pm 0.5}{100} = \pm 0.025$

Error in reading of fourth meter = $1 \times \frac{\pm 1.00}{100} = \pm 0.01$

Fourth meter has least error.

Q.2 The dead zone of a certain pyrometer is 0.125 percent of the span. The calibration is 800°C to 1800°C. What temperature change must occur before it is detected?

Solution:

Given that,

$$\text{Span} = 1800° - 800° = 1000°\text{C}$$

$$\text{Dead zone} = \frac{0.125}{100} \times 1000° = 1.25°\text{C}$$

A change of 1.25°C must occur before it is detected.

Q.3 The limiting errors for a four dial resistance box are:

Units : ±0.2%

Tens : ±0.1%

Hundreds : ±0.05%

Thousands : ±0.02%

If the resistance value is set at 4325 Ω calculate the limiting error for this value.

Solution:

Thousand is set at 4000 Ω and error

$$= \pm 4000 \times \frac{0.02}{100} = \pm 0.8\ \Omega$$

$$\text{For hundred error} = \pm 300 \times \frac{0.05}{100} = \pm 0.15\ \Omega$$

Similarly, $$\text{For ten error} = \pm 20 \times \frac{0.1}{100} = \pm 0.02\ \Omega$$

and $$\text{For unit error} = \pm 5 \times \frac{0.2}{100} = \pm 0.01\ \Omega$$

Hence, $$\text{Total error} = \pm(0.8 + 0.15 + 0.02 + 0.01)\ \Omega = \pm 0.98\ \Omega$$

$$\%\ \text{Relative error} = \frac{0.98}{4325} \times 100 = 0.0226\%$$

Q.4 The following measurement are obtained on a single-phase load:

$$V = 200\text{ V} \pm 1\%,\ I = 5\text{ A} \pm 1\% \text{ and } P = 555\text{ W} \pm 2\%$$

If the power factor is calculated using these measurements. What is the calculated power factor in the worst case error?

Solution:

Given that, Voltage, $V = 220 \pm 1\%$,

Current, $I = 5 \pm 1\%$

Power, $P = 555 \pm 2\%$

$$P = VI \cdot \cos(\phi)$$

⇒ Power factor, $$\text{p.f} = \cos(\phi) = \frac{P}{V.I}$$

$$\text{p.f.} = \cos(\phi) = \frac{555 \pm 2\%}{(220 \pm 1\%)(5 \pm 1\%)} = \frac{555}{220 \times 5} \pm 4\%$$

$$\text{p.f.} = \cos(\phi) = 0.5 \pm 4\%$$

Q.5 An 820 Ω resistance with an accuracy of ±10% carries a current of 10 mA. The current was measured by an analog meter of 25 mA range with an accuracy of ±2% of full scale. Compute the power dissipated in the resistor and determine the accuracy of the result.

Solution:

Resistance, $R = (820 \pm 10\%)\ \Omega$

Current, $I = 10$ mA

Full scale current = 25 mA

Accuracy in current = ±2% of FSD

= ±2% × 25 mA = 0.5 mA

∴ $I = 10\text{ mA} \pm 0.5\text{ mA}$

or $I = (10\text{ mA} \pm 5\%)\text{ mA}$

Power, $P = I^2R$

$$P = (10\text{ mA})^2 \cdot (820) = 0.082\text{ W}$$

Taking log on both sides, $\log P = \log(I^2R)$

Differentiating both sides, $$\frac{\partial P}{P} = 2\frac{\partial I}{I} + \frac{\partial R}{R}$$

$$\therefore \quad \frac{\partial P}{P} = 2 \times 5\% + 10\%$$

$$\frac{\partial P}{P} = 10\% + 10\%$$

$$\frac{\partial P}{P} = 20\%$$

$$\therefore \quad P = 0.082 \text{ W} \pm 20\%$$

Q.6 A variable w is related to three other variables x, y, z as $w = xy/z$. The variables are measured with meters of accuracy ± 0.5% reading, ± 1% of full scale value and ±1.5% reading. The actual readings of the three meters are 80, 20 and 50 with 100 being the full scale value for all three. Find the maximum limiting error in the measurement of variable w.

Solution:

Full scale reading of all three = 100

Readings of x = 80

Readings of y = 20

Reading of z = 50

$$\delta x = \pm 0.5\% \text{ of reading} = \pm\frac{0.5 \times 80}{100} = \pm 0.4$$

$$\delta y = \pm 1\% \text{ of full reading} = \pm\frac{1 \times 100}{100} = \pm 1$$

$$\delta z = \pm 1.5\% \text{ of reading} = \pm\frac{1.5 \times 50}{100} = \pm 0.75$$

Given, $w = \dfrac{xy}{z}$

Taking log, we get

$$\log w = \log x + \log y - \log z$$

Differentiating w.r.t. w we get

$$\frac{\delta w}{w} = \frac{\delta x}{x} + \frac{\delta y}{y} - \frac{\delta z}{z}$$

For maximum limiting error,

$$\frac{\delta w}{w} = \pm\left(\frac{0.4}{80} + \frac{1}{20} + \frac{0.75}{50}\right) \times 100 = \pm 7.00\%$$

Q.7 The following readings were observed when measuring a voltage:

S.No.	1	2	3	4	5	6	7	8
Volts	532	548	543	535	546	531	543	536

Calculate:
(i) Average deviation
(ii) Standard deviation
(iii) Probable error of one reading.

Solution:

Given that:

S.No.	1	2	3	4	5	6	7	8
Volts	532	548	543	535	546	531	543	536

$$\text{Mean} = \frac{532+548+543+535+546+531+543+536}{8} = \frac{2157}{4} = 539.25$$

Deviations,

$$d_1 = 532 - 539.25 = -7.25$$
$$d_2 = 548 - 539.25 = 8.75$$
$$d_3 = 543 - 539.25 = 3.75$$
$$d_4 = 535 - 539.25 = -4.25$$
$$d_5 = 546 - 539.25 = 6.75$$
$$d_6 = 531 - 539.25 = -8.25$$
$$d_7 = 543 - 539.25 = 3.75$$
$$d_8 = 536 - 539.25 = -3.25$$

(i) Average deviation, $\bar{d}_{avg.} = \dfrac{|d_1|+|d_2|+|d_3|+|d_4|+|d_5|+|d_6|+|d_7|+|d_8|}{8} = 5.75$

(ii) Standard deviation, $\sigma = \sqrt{\dfrac{(d_1)^2+(d_2)^2+(d_3)^2+(d_4)^2+(d_5)^2+(d_6)^2+(d_7)^2+(d_8)^2}{n-1}}$

$$= \sqrt{\frac{299.5}{7}} = \sqrt{42.7857} = 6.541$$

(iii) Probable error $= 0.6745\,\sigma = 0.6745 \times 6.541 = 4.4119$

Q.8 **Two resistors R_1 and R_2 are connected in series and then in parallel. The values of resistance are:**

$R_1 = 100.0 \pm 0.1\ \Omega$; $R_2 = 50 \pm 0.03\ \Omega$

Calculate the uncertainty in the combined resistance for both series and parallel arrangements.

Solution:

When the two resistances are connected in series the resultant resistance is

$$R = R_1 + R_2$$

$$\frac{\partial R}{\partial R_1} = 1 \quad \text{and} \quad \frac{\partial R}{\partial R_2} = 1$$

Hence uncertainty in the total resistance is

$$\boxed{W_R = \pm\sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 W_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 W_{R_2}^2}}$$

$$W_R = \pm\sqrt{(1)^2 \times (0.1)^2 + (1)^2 \times (0.03)^2} = \pm 0.1044\ \Omega$$

The total resistance is $R = 100 + 50 = 150\ \Omega$

and can be expressed as $R = 150 \pm 0.1044\ \Omega$

When the two resistances are connected in parallel the resultant resistance is

$$R = \frac{R_1 R_2}{(R_1 + R_2)} = \frac{100 \times 50}{100 + 50} = 33.3333\ \Omega$$

$$R = \frac{R_1 R_2}{(R_1 + R_2)}$$

$$\frac{\partial R}{\partial R_1} = \frac{R_2(R_1+R_2) - R_1 R_2(1)}{(R_1+R_2)^2} = \frac{R_2}{R_1+R_2} - \frac{R_1 R_2}{(R_1+R_2)^2} = \frac{50}{150} - \frac{100 \times 50}{(150)^2} = 0.1111$$

$$\frac{\partial R}{\partial R_2} = \frac{R_1(R_1+R_2) - R_1 - R_2(1)}{(R_1+R_2)^2} = \frac{R_1}{R_1+R_2} - \frac{R_1 R_2}{(R_1+R_2)^2}$$

$$= \frac{100}{150} - \frac{100\times 50}{(150)^2} = 0.4444$$

Hence uncertainty in total resistance is

$$W_R = \pm\sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 W_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 W_{R_2}^2} = \pm\sqrt{(0.1111)^2\times(0.1)^2 + (0.4444)^2\times(0.03)^2} = \pm\, 0.01735\ \Omega$$

The total resistance can be written as,

$$R = 33.3333 \pm 0.01735\ \Omega$$

Q.9 The law of deflection of a galvanometer is $I = k\theta/\cos\theta$, where I is the current, k is a constant and θ is the deflection. If the angle of deflection θ is known to be within ±0.1° (standard deviation) of 15°, what is the percentage standard deviation of current, I?

Solution:

Given that, $$I = \frac{k\theta}{\cos\theta} = k\theta(\cos\theta)^{-1}$$

$\therefore$ $$\frac{\partial I}{\partial\theta} = k[(\cos\theta)^{-1} + \theta(\cos\theta)^{-2}\sin\theta]$$

Put, $\theta = 15°$

$$= k\left[\frac{1}{0.966} + \frac{\pi}{180}\times 15\times\frac{1}{(0.966)^2}\times 0.259\right] = 1.108\ k$$

Standard deviation of I is $\sigma_I = \dfrac{\partial I}{\partial\theta}\sigma_\theta$

$$= \pm(1.108\ k)\left(0.1\times\frac{\pi}{180}\right)\text{rad}$$

Percentage standard deviation of I is

$$= \frac{\sigma_I}{I}\times 100 = \pm\frac{(1.108\ \text{k})\left(0.1\times\frac{\pi}{180}\right)}{k\times\frac{\pi}{180}\times 15°\times(\cos 15°)^{-1}}\times 100 = \pm 0.7135\%$$

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ELECTROMAGNETIC THEORY

CONVENTIONAL PRACTICE SETS

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1 CHAPTER

Vector Analysis

Q.1 Given point, $P(9, -12, 15)$ is in Cartesian system. Express P in cylindrical and spherical systems.

Solution

$$\rho = \sqrt{x^2+y^2} = \sqrt{(9)^2+(-12)^2} = 15$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\left[\frac{(-12)}{9}\right] = -53.13°$$

P is cylindrical format will be as, $P = (15, -53.13°, 15)$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{(9)^2+(-12)^2+(15)^2} = 21.213$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = \tan^{-1}\left[\frac{15}{15}\right] = 45°$$

P in spherical format will be as, $P = (21.213, 45°, -53.13°)$

Q.2 Let $\vec{H} = 5\rho\sin\phi\,\hat{a}_\rho - \rho z\cos\phi\,\hat{a}_\phi + 2\rho\hat{a}_z$ A/m. At point $P(2, 30°, -1)$ find:

(a) a unit vector along $\vec{H}$.

(b) the component of $\vec{H}$ parallel $\hat{a}_x$.

(c) the component of $\vec{H}$ normal to $\rho = 2$.

(d) the component of $\vec{H}$ tangential to $\phi = 30°$.

Solution:

At P, $\rho = 2, \phi = 30°, z = -1$

$$\vec{H} = 10\sin 30°\,\hat{a}_\rho + 2\cos 30°\,\hat{a}_\phi - 4\hat{a}_z = 5\hat{a}_\rho + 1.732\,\hat{a}_\phi - 4\hat{a}_z \text{ A/m}$$

(a) Unit vector along $\vec{H}$,

$$\hat{a}_H = \frac{5\hat{a}_\rho + 1.732\,\hat{a}_\phi - 4\hat{a}_z}{\sqrt{5^2+1.732^2+4^2}} = 0.7538\,\hat{a}_\rho + 0.2611\hat{a}_\phi - 0.603\hat{a}_z$$

(b) $H_x = H_\rho\cos\phi - H_\phi\sin\phi = 5\rho\sin\phi\cos\phi - \rho z\cos\phi\sin\phi$

or, P at $\rho = 2, \quad \phi = 30°, z = -1$

$$H_x = H_x\,\hat{a}_x = (10\sin 30°\cos 30° + 2\sin 30°\cos 30°)\,\hat{a}_x = 5.196\,\hat{a}_x \text{ A/m}$$

(c) Normal to $P = 2$ is $\vec{H}_\rho = \vec{H}_\rho\,\hat{a}_\rho$

i.e. $\vec{H}_n = 0.7538\,\hat{a}_\rho$ A/m

(d) Tangential to $\phi = 30°$

$$H_t = H_\rho\,\hat{a}_\rho + H_z\,\hat{a}_z = 0.7538\,\hat{a}_\rho - 0.603\,\hat{a}_z \text{ A/m}$$

Q.3 E and F are vector fields given by $\vec{E} = 2x\,\hat{a}_x + \hat{a}_y + yz\hat{a}_z$ and $\vec{F} = xy\,\hat{a}_x - y^2\hat{a}_y + xyz\hat{a}_z$. Determine:

(a) $|E|$ at (1, 2, 3)

(b) The component of $\vec{E}$ along $\vec{F}$ at (1, 2, 3)

(c) A vector perpendicular to both $\vec{E}$ and $\vec{F}$ at (0, 1, –3) whose magnitude is unity.

Solution:

(a)
$$\vec{E} = 2x\,\hat{a}_x + \hat{a}_y + yz\,\hat{a}_z$$

At point (1, 2, 3) $\Rightarrow$
$$\vec{E} = 2\hat{a}_x + \hat{a}_y + 6\hat{a}_z$$
$$|\vec{E}| = \sqrt{2^2 + 1^2 + 6^2} = \sqrt{41} = 6.403$$

(b)
$$\vec{F} = xy\,\hat{a}_x - y^2\,\hat{a}_x + xyz\,\hat{a}_z$$

At (1, 2, 3),
$$\vec{F} = 2\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z$$

$\therefore$ The component of $\vec{E}$ along $\vec{F}$

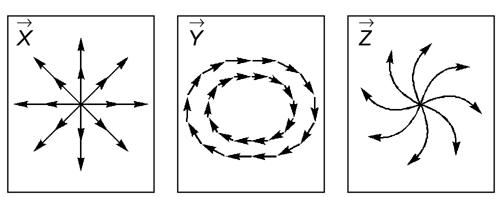
$$\vec{E}_F = (\vec{E}\cdot a_F)\hat{a}_F = \frac{(\vec{E}\cdot\vec{F})}{|\vec{F}|}\bar{a}_F = \frac{36}{56}(2\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z) = 1.286\hat{a}_x - 2.571\hat{a}_y + 3.857\hat{a}_z$$

(c) At (0, 1, –3)
$$\vec{E} = 0\hat{a}_x + \hat{a}_y - 3\hat{a}_z$$
$$\vec{F} = 0\hat{a}_x - \hat{a}_y + 0\hat{a}_z$$

$$E\times\vec{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = -3\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z$$

$$a_{E\times F} = \pm\frac{\vec{E}\times\vec{F}}{|\vec{E}\times\vec{F}|} = \pm\hat{a}_x$$

Q.4 The figures show diagrammatic representations of vector fields $\vec{X}$, $\vec{Y}$ and $\vec{Z}$, respectively. What can you comment about these diagrams?



Solution:

$\vec{X}$ is going away so $\vec{\nabla}\cdot\vec{X} \neq 0$.

$\vec{Y}$ is moving circulator direction so $\vec{\nabla}\times\vec{Y} \neq 0$.

$\vec{Z}$ has circular rotation so $\vec{\nabla}\times\vec{Z} \neq 0$.

Q.5 Find the divergence of vector field, $\vec{V}(x,y,z) = -(x\cos xy + x)\hat{i} + (y\cos xy)\hat{j} + (\sin z^2 + x^2 + y^2)\hat{k}$.

Solution:

$$\vec{V}(x, y, z) = -(x\cos xy + y)\hat{i} + (y\cos xy)\hat{j} + [\sin(z)^2 + (x^2) + (y^2)]\hat{k}$$

$$\text{Divergence} = \nabla\cdot V$$
$$= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$
$$= -\cos xy + x(\sin xy)y + \cos xy - y\sin(xy)x + 2z\cos z^2 = 2z\cos z^2$$

Q.6 "A hand 'curl meter' in the form of a pin wheel is used to indicate curl of a vector field." Justify the statement.

Solution:

A pin or paddle wheel as a 'curl meter': The force is exerted on the each blade of the paddle wheel, the force being proportional to the component of the field normal to the surface of that blade. To test and field for curl we dip one paddle wheel into the field with the axis of the paddle wheel lined up with the direction of the component of curl desired. No rotation means no curl, larger angular velocities mean greater values of the curl a reverse in the direction of spin means a reversal in the sign of the curl. In order find the direction of vector curl, we should place one paddle wheel in the field and hunt around for the orientation which produces the greatest torque. The direction of the curl is then along the axis of the paddle wheel.

Q.7 Consider a function $\vec{f} = \frac{1}{r^2}\hat{r}$, where r is the direction from origin and $\hat{r}$ is unit vector in radial direction.

Find divergence of this function over a sphere of radius R, which includes origin.

Solution:

$$\vec{f} = \frac{1}{r^2}\cdot\hat{r}$$

From divergence theorem as we know,

$$\boxed{\int_{\text{vol.}}(\nabla\cdot\vec{f})dV = \oiint_S \vec{f}\cdot d\vec{S}}$$

$$\oiint_S \vec{f}\cdot d\vec{S} = \oiint_S\left(\frac{1}{r^2}\cdot\hat{r}\right)\cdot\left(r^2\sin\theta\cdot d\theta\cdot d\phi\cdot\hat{r}\right) = \oiint_S \sin\theta\cdot d\theta\cdot d\phi = 4\pi$$

Q.8 For a vector field $\vec{A}$, show explicitly that $\nabla\cdot\nabla\times\vec{A} = 0$; that is, the divergence of the curl of any vector field is zero.

Solution:

To shows $\vec{\nabla}\cdot\vec{\nabla}\times\vec{A} = 0$, let us assume that $\vec{A}$ is a function of x, y, z then,

$$\vec{\nabla}\cdot\vec{\nabla}\times\vec{A} = \vec{\nabla}\cdot\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}$$

$$= \vec{\nabla}\cdot\left[\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\hat{x} - \left(\frac{\partial A_z}{\partial x}-\frac{\partial A_x}{\partial z}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\hat{z}\right]$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial A_z}{\partial x}-\frac{\partial A_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)$$

$$= \frac{\partial^2 A_z}{\partial x\partial y} - \frac{\partial^2 A_y}{\partial x\partial z} - \frac{\partial^2 A_z}{\partial y\partial x} + \frac{\partial^2 A_x}{\partial y\partial z} + \frac{\partial^2 A_y}{\partial z\partial x} - \frac{\partial^2 A_x}{\partial z\partial y} = 0$$

So it is proved that divergence of curl of any vector field is always 0.

Q.9 Find the rate at which the scalar function, $V = r^2\sin 2\phi$, in cylindrical co-ordinates, increases in the direction of the vector $\vec{A} = \hat{a}_r + \hat{a}_\phi$ at the point having co-ordinates $(2, \pi/4, 0)$.

Solution:

As we know that, Gradient is a vector that represents both the magnitude and the direction of maximum space rate of the increase of the scalar function, i.e.,

$$\text{grad } V = \nabla V = \frac{dV}{dn}\hat{a}_n \quad \text{...(i)}$$

But in cylindrical coordinate system, the grad V can be defined as,

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z \quad \text{...(ii)}$$

For the case under consideration, the quantity required is,

$$\nabla V \cdot \hat{a}_A = \nabla V \cdot \frac{A}{|A|} \quad \text{at } (2, \pi/4, 0)$$

From equation (ii) we have,

$$\nabla V = \frac{\partial}{\partial r}(r^2 \sin 2\phi)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \phi}(r^2 \sin 2\phi)\hat{a}_\phi + \frac{\partial}{\partial z}(r^2 \sin 2\phi)\hat{a}_z.$$

or, $$\nabla V = 2r\sin 2\phi\ \hat{a}_r + 2r\cos 2\phi\ \hat{a}_\phi$$

Now, $$\nabla V \cdot A = (2r\sin 2\phi\ \hat{a}_r + 2r\cos 2\phi\ \hat{a}_\phi)\cdot(\hat{a}_r + \hat{a}_\phi)$$

or, $$\nabla V \cdot A = 2r\sin 2\phi + 2r\cos 2\phi \quad \text{...(iii)}$$

Also, $$|\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$\therefore$ $$\nabla V \cdot \hat{a}_A = \frac{\nabla V \cdot A}{|A|} = \frac{1}{\sqrt{2}}[2r\sin 2\phi + 2r\cos 2\phi] = (\sqrt{2}\, r\sin 2\phi + \sqrt{2}\, r\cos 2\phi)$$

Now, $$(\nabla V \cdot \hat{a}_A)_{at(2,\pi/4,0)} = \sqrt{2}\times 2\times \sin 2\times\frac{\pi}{4} + \sqrt{2}\times 2\times\cos 2\times\frac{\pi}{4} = 2\sqrt{2}$$

Q.10 A non-uniform field is $\vec{E} = y\hat{a}_x + x\hat{a}_y + 2\hat{a}_z$ V/m. Find the work experienced in carrying 2C charge from point $B(1, 0, 1)$ to A (0.8, 0.6, 1) along the shortest arc to the circle $x^2 + y^2 = 1$, $z = 1$.

Solution:

In Cartesian coordinate system, the differential path is

$$d\vec{L} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

Therefore, the work done in moving charge from B to A is

$$\boxed{W = -Q\int_B^A \vec{E}.d\vec{L}}$$

$$W = -2\int_B^A (y\hat{a}_x + x\hat{a}_y + 2\hat{a}_z)(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z)$$

$$= -2\int_{x=1}^{x=0.8} y\,dx - 2\int_{y=0}^{y=0.6} x\,dy - 4\int_{z=1}^{z=1} dz = -2\int_{x=1}^{x=0.8}\sqrt{1-x^2}\,dx - 2\int_{y=0}^{y=0.6}\sqrt{1-y^2}\,dy - 0$$

Let, $$x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$$

Similarly, $$y = \sin\theta \Rightarrow dy = \cos\theta\, d\theta$$

$$= -2\int_{90°}^{53.13°}\sqrt{1-\sin^2\theta}.\cos\theta d\theta - 2\int_{0°}^{36.869°}\sqrt{1-\sin^2\theta}.\cos d\theta$$

$$= -2\int_{90°}^{53.13°}\cos^2\theta d\theta - 2\int_{0°}^{36.869°}\cos^2\theta.d\theta = 0.1635 - 1.12339 = -0.96 \text{ J}$$